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FINAL REPORT

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Published Papers

1) "Motion of a bore over a sloping beach"; H. B. Keller, D. A. Levine, G. B. Whitham; Journal of Fluid Mechanics, Vol. 7, Part 2, pp. 302-316, 1960.

2) "On the propagation of cylindrical shock waves"; Akira Sakurai; Exploding Wires, Plenum Press, N. Y., 1959; Chace and More, editors; pp. 264-270.

Papers Accepted for Pulbication

1) "An improved perturbation theory for shock waves propagating through non-uniform regions"; M. P. Friedman; to appear in the Journal of Fluid Mechanics.

2) "On the problem of a shock wave arriving at the edge of a gas"; Akira Sakurai; to appear in the Communications on Pure and Applied Mathematics.

Further Work

1) Propagation of Shock Waves Through Non-uniform Regions.

(H. B. Keller, D. A. Levine, D. Loveland)

The numerical procedure developed in the paper "Motion of a bore over a sloping beach" has been extended for application to the problem of shocks moving through non-uniform ducts. The finite difference equations employ a fixed Eulerian net in the (x,t) plane and an iterative method for fitting in the shock at each time increment. This method has many advantages over previously employed characteristic net calculations. In addition, the stability and convergence of the method have been demonstrated and sufficient conditions for the convergence of the shock-fitting iterations have been determined.

A general code which employs this numerical procedure has been written for the I.B.M. 704. The channel area in this program can be of the general form:

$$A(x) = 1, \quad x \leq 0; \quad A(x) = \text{arbitrary function of } x, \quad x > 0 .$$

Initially (at $t = 0$) a shock can be at any position $x = \xi(0)$ and is assumed propagating in the positive x-direction with a uniform gas ahead of the shock (i.e. in $x > \xi(0)$). The gas is assumed polytropic with arbitrary γ (> 1).

Test calculations are being performed at the present time. When the code is completely debugged it is planned that a series of calculations with varying channel shapes will be run. If the results show significant improvement over previous calculations or if novel shapes are included a paper will be written describing

this material in detail. Due credit will be given to this contract in any subsequent publications related to this work.

2) Structure of Magnetohydrodynamic Shock Waves. (G. B. Whitham)

The discussion of this topic contained in "Some comments on wave propagation and shock wave structure" (G. B. Whitham, Comm. Pure Appl. Math., Feb. 1959) has been extended to include a normal magnetic field and oblique shocks. One interesting result is the non-uniqueness of the shock profile in certain cases. This result was noted previously by G. S. S. Ludford in "The structure of a hydromagnetic shock in steady plane motion" (J.F.M., 1959). Ludford claims that all but one of the profiles can be eliminated as being unrealistic. However, we claim that his arguments are wrong and that the lack of uniqueness is genuine. Plausible arguments are provided to explain why the profile is not unique in these cases.

Some further results have been obtained on this problem. However, it has recently been discovered that P. Germain has considered such problems and previously obtained some of our results. Thus it is not clear that a paper should be written on this work but due credit will be given to this contract in the event that such a paper is published.

3) Three-shock Intersection. (Akira Sakurai, J. Hesse, H. B. Keller)

Dr. Sakurai has proposed new ideas to resolve the discrepancy between the simple three-shock theory of Mach reflection and

experiments. He introduces viscous effects in the sector which contains the vortex sheet, on the grounds that the discontinuities are weak in the cases where the theory does not check.

A rather complete numerical solution of Dr. Sakurai's theory has been obtained. These results indicate better agreement with the experiments than does the simple three-shock theory. This work has been described in an unpublished paper. There is still some doubt as to the validity of the theory and until these doubts can be eliminated no report will be issued. If such a paper or report is published this contract will receive due credit.

4) Boundary-layer Transition to Turbulence. (D. F. De Santo, H. B. Keller)

This work is a rather complete revision of some studies initiated by H. F. Ludloff under A.F.O.S.R. - A.R.D.C. Contract AF 18(603)-25. The problem is to derive from the time dependent, viscous equations of motion some of the phenomena that occur during the transition from laminar to turbulent flow along a flat plate. It is assumed that well within the boundary layer a small harmonic disturbance is present and the deviations from the Blasius flow are sought. However, no linearizations are employed. In terms of the vorticity, ω, and stream function, ϕ, the Navier-Stokes equations reduce to a non-linear parabolic partial differential equation coupled with a Poisson equation. The domain of interest is chosen as some sufficiently large rectangle downstream from the disturbance.

In this rectangle the equations of motion are approximated by appropriate difference equations and numerical solutions are sought. The difference equations have been shown to be convergent and stable, under a mild restriction on the net spacing. (It is believed that the method is unconditionally stable, but this has not yet been demonstrated.) The difference form of the Poisson equation is solved by a rapidly converging iteration while the semi-implicit parabolic difference equations are solved explicitly.

Some test calculations have been performed for small amplitude (almost sonic) disturbances. However, a high frequency wave develops and grows, and its cause has been traced to the treatment of the boundary conditions on the upstream boundary of the rectangle. At the present this difficulty is being investigated and some methods to eliminate it have been proposed. Further calculations are planned and this contract will be credited in any subsequent publications resulting from this work.

5) Solar Spicules. (H. B. Keller, R. Lüst, A. Lapidus)

This problem is concerned with the formation and focusing of magnetohydrodynamic shocks in a solar chromosphere. One of the basic objectives is to devise and test numerical procedures for treating such problems in two space dimensions. In the particular problem under study a perfectly conducting gas (Lundquist model) initially in equilibrium under gravity, with a vertical magnetic field present, is subjected to a transient localized (pressure) pulse near the bottom surface (assumed rigid).

The numerical procedure used is a modification of Lax's conservation form for the difference equations. Artificial boundaries must be introduced, due to the limitations of existing computing equipment, and have the effect of causing some reflections of any waves incident on them. The treatment at these boundaries has been modified several times in order to reduce the effect of such reflections. When the magnetic field is absent (i.e. ordinary hydrodynamics) the procedure works quite well showing the formation of well defined shocks which propagate as expected. If gravity is absent the results are even better (since uniform orders of magnitude are then maintained in the ambient data) and comparisons can be made with accurate (but approximate) analytical solutions.

However, when both magnetic field and gravitational effects are present some difficulties remain. Shocks can be seen to form and are even focused along the field but spurious disturbances still arise from the top boundary. These signals grow as they propagate downward; this is to be expected for the magnetohydrodynamic "fast waves". Whether these spurious effects can be tolerated or not depends upon their relative amplitude and how far they propagate before the relevant phenomena have occurred. Methods for reducing these disturbances are being considered and further computations are in progress.

6) Secondary Shocks Within Spherical and Cylindrical Blasts.

(M. P. Friedman)

Investigations into the behavior of the gas flow behind spherical or cylindrical blasts have shown the presence of a

secondary shock. This secondary shock, at first weak, is initially carried outward with the expanding gases; as it develops it strengthens and then bends back toward the origin arriving there with great intensity. Descriptions of the flow field due to a spherical or cylindrical detonation can be obtained by numerically integrating the complete equations of motion (H. L. Brode, Phys. of Fl., April 1959). However, using some recently developed techniques in shock dynamics (R. F. Chisnell, J. Fl. Mech., 1957; G. B. Whitham, Comm. Pure and Appl. Math., 1952) it is possible to obtain explicit formulae describing the main blast wave and the formation and subsequent motion of the secondary shock. The main blast wave is described by using Chisnell's shock-area rule while the secondary shock is obtained initially by the Whitham weak shock approximation. When the secondary shock is strengthened beyond the region of validity of the weak shock approximation a generalized shock-area rule is used, the generalization being in adapting it to the case of a non-uniform flow ahead of the shock.

These formulae are now being programmed for the I.B.M. 704 computer and a test problem, which has been solved by numerical integration of the flow equations (Brode) will be used to check out the present theory. Any subsequent publications resulting from this work will be credited to this contract.

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